Tabu Search: A Tutorial

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Tabu search: Introduction

A higher level heuristic procedure for solving optimization problems, designed to guide other methods to escape the trap of local optimality

Flexible structured memory is used to permit search information to be exploited more thoroughly than by rigid memory systems

Strategically constraining and freeing the search process embodied in ***tabu restrictions*** and ***aspiration criteria***

Memory functions of varying time spans are used for ***intensifying*** and ***diversifying*** the search

Overview: Three primary themes

Use of **flexible attribute-based memory structures**

- Designed to permit evaluation criteria and historical search information
- Exploited more thoroughly than by rigid memory structures (as in branch and bound and A* search) or by memoryless systems (as in simulated annealing and other randomized approaches)

Overview: Three primary themes

An associated mechanism of control

- For employing the memory structures
- Based on the interplay between conditions that constrain (*tabu restriction*) and free the search process (*aspiration criteria*)

The incorporation of memory functions of different time spans

- From short term to long term, to implement strategies for intensifying and diversifying the search
- *Intensification strategies* reinforce move combinations and solution features historically found good
- *Diversification strategies* drive the search into new regions

Short-term Memory and Aggressive Search

The short-term memory constitutes a form of aggressive exploration

Make the best move possible under tabu restrictions: Figure 1

Tabu restrictions are designed to prevent the reversal, or sometimes repetition of certain moves by rendering selected attributes of these moves forbidden (tabu)

The primary goal of the tabu restriction is to permit the method to go beyond points of local optimality

Tabu restrictions are intended to prevent cycling (revisiting some earlier solution) behavior and to induce the search to follow a new trajectory if cycling occurs

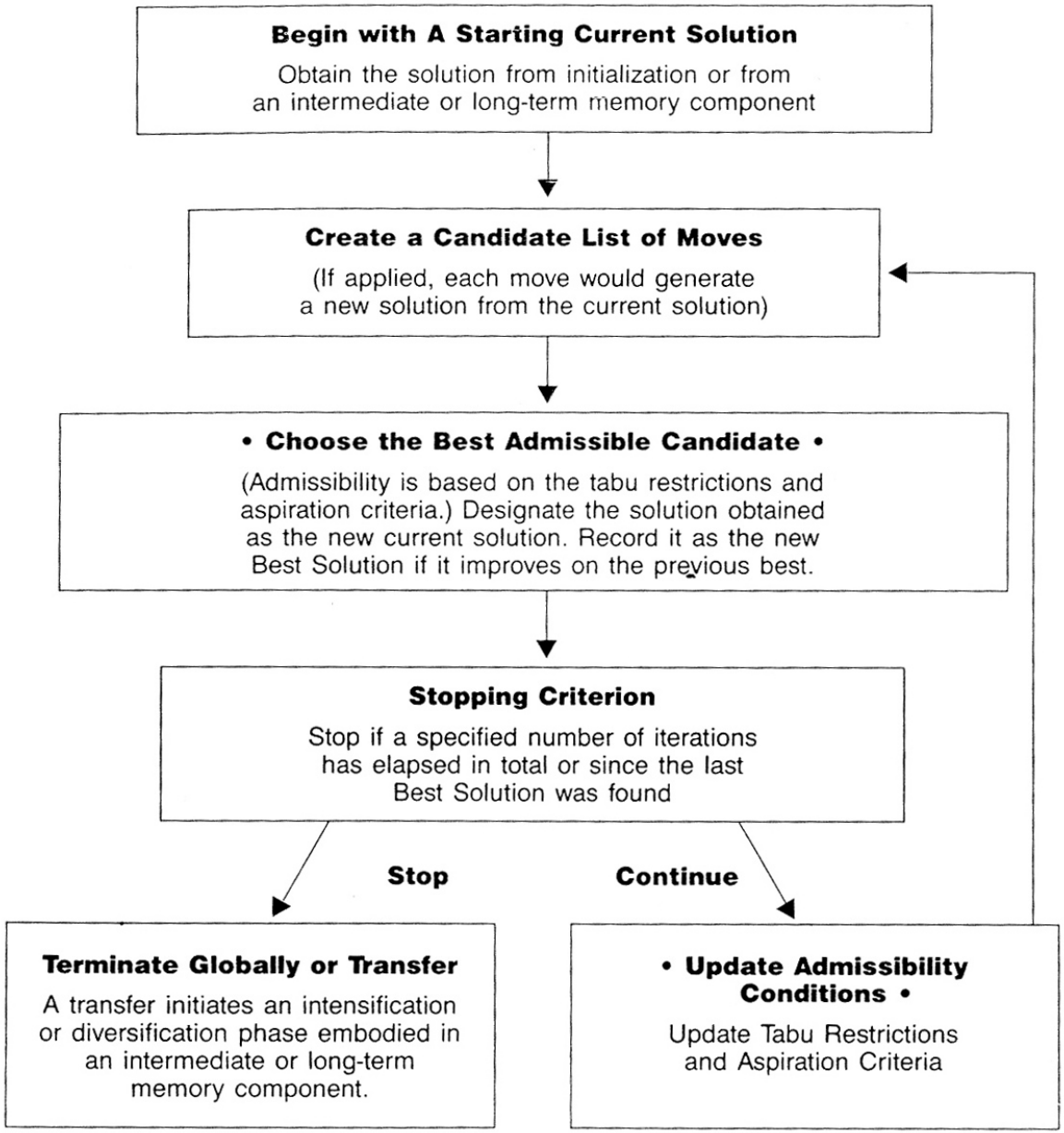


Figure 1: Tabu search short-term memory component.

Determining the Best Candidate

Aggressive orientation of short-term memory chooses the best admissible candidate: Figure 2

1. Each of the moves of the candidate list is evaluated in turn
2. Check tabu status for admissibility
3. If the move is tabu, the aspiration criteria are given as an opportunity to override the tabu status

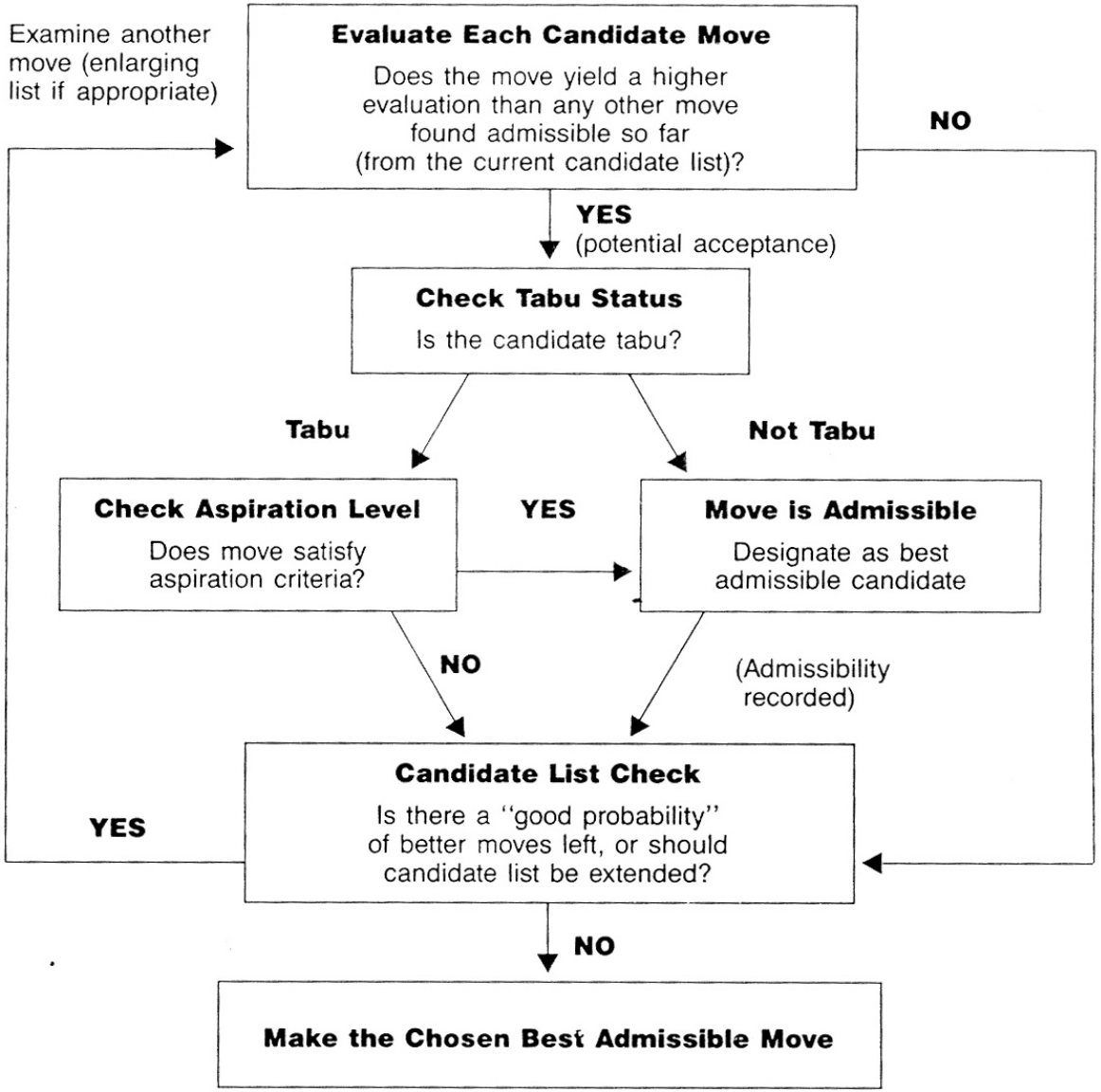


Figure 2: Selecting the best admissible candidate.

A Simple Illustration of Tabu Search

Constrained minimum cost spanning tree problem: Figure 3

"edge swap" move

Tabu restriction - forbids a future move to drop the edge as long as it remains tabu, permits only two edges to be tabu at any given time, i.e., each added edge remains tabu for two iterations

Aspiration criterion - allows the current move to include a tabu edge if the resulting tree is better than the best tree produced so far

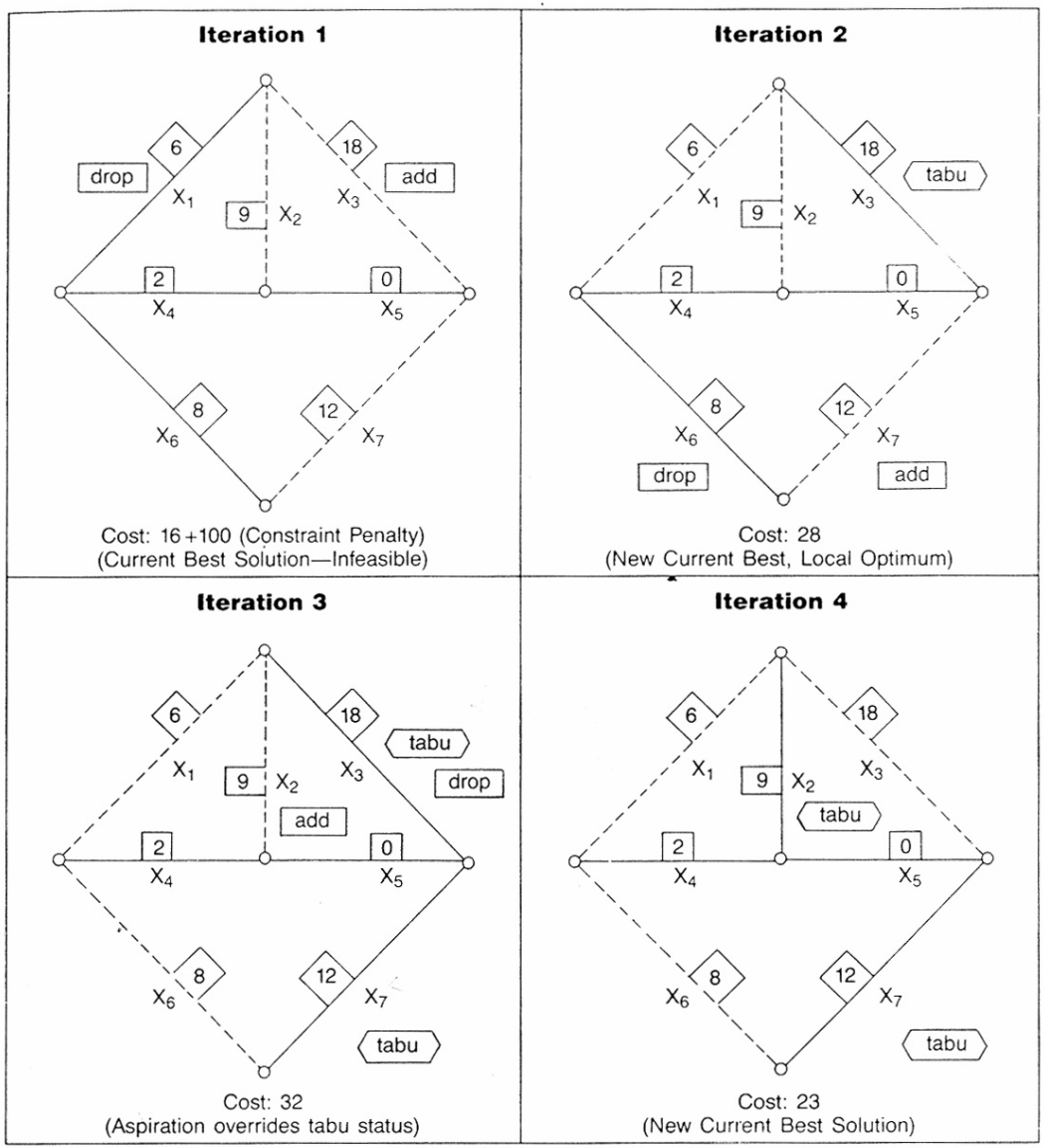


Figure 3: Illustrated solution: minimum-cost trees. The choice rule is select the least-cost admissible "edge swap." The tabu restriction is forbid dropping one of the two most recently added edges (these edges are designated tabu). The aspiration criterion is override the tabu restriction if the swap produces a new "current best solution." (Constraints: $X_1 + X_2 + X_6 \leq 1$, $X_1 \leq X_3$. Violation penalty = 50.)

Preliminary Guidelines

1. Tabu restriction

Prevent added edges from being dropped

Prevent dropped edges from being added

Guideline 1: Tabu restriction based on single type of move attribute - select an attribute whose tabu status less rigidly restricts the choice of available moves

Recommended tabu list size: 5-12

The more stringent the tabu restriction, the smaller the tabu list size

Guideline 2: Incorporate separate but parallel tabu lists for different attribute types

i.e., add-list, drop-list

Preliminary Guidelines

2. Aspiration Criteria

Each selected attribute of a move can have one or more aspiration criteria of its own

Aspiration criteria can be made time dependent

Guideline 3: Embody the treatment of aspiration criteria in an attribute-based framework

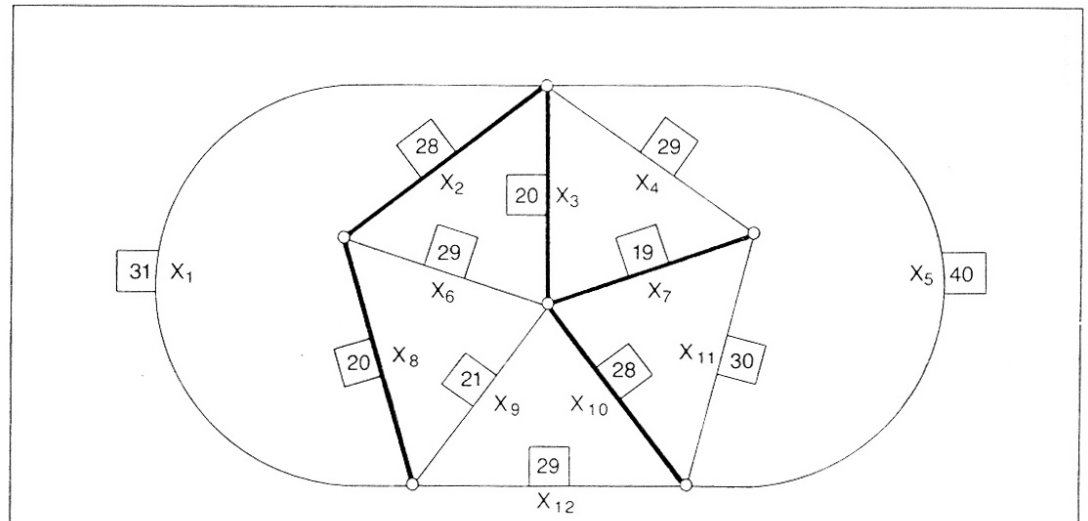
Preliminary Guidelines

3. Intermediate and Long-term Memory: Intensification and Diversification Tradeoffs

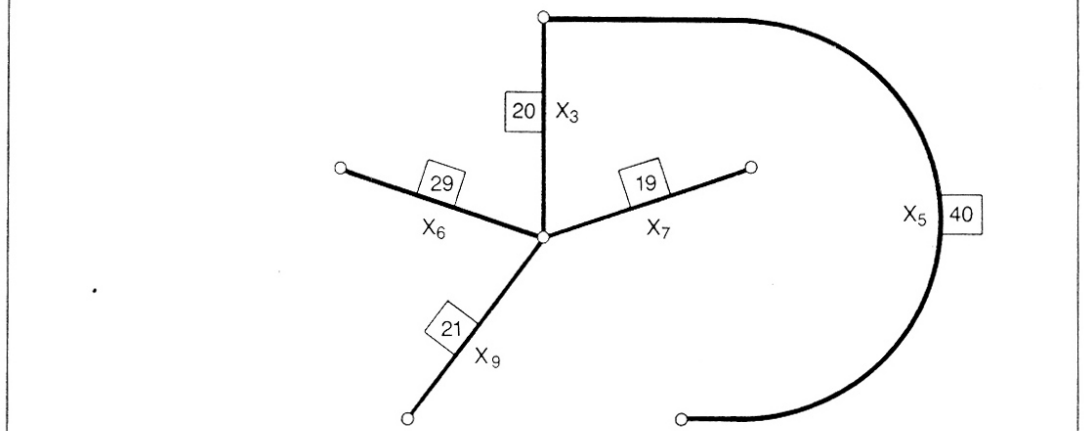
Long-term memory is important for obtaining best results for hard problems

Example: Figure 4

The tabu search procedure that relies only on the short-term memory component fails to discover the right move to reach the optimal solution



Initial Solution: Cost =115 +100 (Penalty)



Optimal Solution: Cost =129

Figure 4: Relevance of longer-term memory and diversification minimum-cost tree problem. Added constraints are $X_9 \leq X_7$, $X_3 + X_7 \leq 2X_5$. (Unit violation penalty = 50.)

Preliminary Guidelines

4. Links to a Learning Process

Target analysis [Glover and Greenberg, 1989]: A learning approach which offers a useful means for developing evaluators to support the intensification and diversification strategies of tabu search

Guideline 4: To combine the diversification and intensification goals, create a rating system by reference to target analysis, maintaining records of highly rated attributes and of how often these attributes appear in solutions generated

Preliminary Guidelines

Implementation of Guideline 4: An example

Move attributes may be divided into six frequency classes

(1) often occur in good solutions

(2) often occur in poor solutions but rarely in good solutions

(3) often occur in moves to add the attribute to the current solution, where these moves receive evaluations that are high, but not high enough to be chosen

(4) often occur in moves to drop the attribute from the current solution, where these moves receive evaluations insufficiently attractive to be chosen

Preliminary Guidelines

(5) often occur in the solutions actually generated during the search process (whether good or bad)

(6) often do not occur in solutions generated

Class (1) and (2): used to support intensification

Class (3) and (4): combine the elements of intensification and diversification

Class (5) and (6): emphasize diversification

Preliminary Guidelines

5. Diversification Based on Move Distance

Selecting preferred moves from infrequently sampled classes provides a useful form of diversification

Moves that induce greater solution changes should be applied when standard evaluators lose their effectiveness

Guideline 5: Devote a limited number of preliminary iterations to identify historical statistics for move evaluations in each distance class, and apply relationships from target analysis to obtain effective choices at subsequent iterations